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Grand Unification on Noncommutative Spacetime

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Abstract

We compute the beta-functions of the standard model formulated on a noncommutative spacetime. If we assume that the scale for spacetime noncommutativity is of the order of 8.2×10^{12} GeV we find that the three gauge couplings of the standard model merge at a scale of 2.3×10^{17} GeV. The proton lifetime is thus much longer than in conventional unification models.

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Grand unification [1–3] is a topic that has fascinated theoretical physicists since the discovery of the standard model which is based on the gauge symmetry SU(3)×SU(2)×U(1). It is tempting to try to unify these groups within a bigger group such as SU(5) [1] or SO(10) [2]. Unfortunately the gauge couplings of the standard model fail to converge to one unified gauge coupling [4] unless one plays with threshold effects [5] or breaks the fundamental symmetry of the grand unified gauge group in different steps (see e.g. [6]). Another way to obtain the unification of the gauge couplings of the standard model is to introduce new particles, e.g. supersymmetric particles (see e.g. [7]), to reach the numerical unification of the gauge couplings. In this letter we shall pursue a different approach and study whether spacetime noncommutativity can modify the standard model in such a way that the gauge couplings converge to one unified gauge coupling. We do not introduce any new particles and consider a direct breaking of the grand unified gauge symmetry to the standard model.

Gauge theories formulated on a canonical noncommutative spacetime have recently received lots of attention (see e.g. [8,9]). A canonical noncommutative spacetime is defined by the noncommutative algebra

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \tag{1}$$

where μ and ν run from 0 to 3 and where $\theta^{\mu\nu}$ is constant and antisymmetric. It has mass dimension minus two. Formulating Yang-Mills theories relevant to particle physics on such a spacetime requires to consider matter fields, gauge fields and gauge transformations in the enveloping algebra otherwise SU(N) gauge symmetries cannot be implemented [10, 11] . The enveloping algebra approach allows to map a noncommutative action $\hat{S}$ on an effective action formulated on a regular commutative spacetime.

We shall be working within the framework of the minimal noncommutative standard model [11]. This model is minimal in the sense that it has minimal deviations with respect to the standard model on a commutative spacetime. It can be written in a very compact way:

$$\begin{aligned}
\hat{S}_{NCSM} \quad = \quad & \int d^4x \sum_{i=1}^{3} \overline{\widehat{\Psi}}^{(i)}_L \star i\widehat{\not{D}}\widehat{\Psi}^{(i)}_L + \int d^4x \sum_{i=1}^{3} \overline{\widehat{\Psi}}^{(i)}_R \star i\widehat{\not{D}}\widehat{\Psi}^{(i)}_R \\
& - \int d^4x \frac{1}{2g'} \mathbf{tr_1} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} - \int d^4x \frac{1}{2g} \mathbf{tr_2} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} \\
& - \int d^4x \frac{1}{2g_3} \mathbf{tr_3} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} + \int d^4x \Big(\rho_0(\widehat{D}_\mu \widehat{\Phi})^\dagger \star \rho_0(\widehat{D}^\mu \widehat{\Phi}) \\
& - \mu^2 \rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi}) - \lambda \rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi}) \star \rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi}) \Big) \\
& + \int d^4x \Big(- \sum_{i,j=1}^{3} W^{ij} \Big((\bar{\widehat{L}}^{(i)}_L \star \rho_L(\widehat{\Phi})) \star \widehat{e}^{(j)}_R + \bar{\widehat{e}}^{(i)}_R \star (\rho_L(\widehat{\Phi})^\dagger \star \widehat{L}^{(j)}_L) \Big)
\end{aligned} \tag{2}$$

$$- \sum_{i,j=1}^{3} G_u^{ij} \Big((\bar{\widehat{Q}}_L^{(i)} \star \rho_{\bar{Q}}(\widehat{\bar{\Phi}})) \star \widehat{u}_R^{(j)} + \bar{\widehat{u}}_R^{(i)} \star (\rho_{\bar{Q}}(\widehat{\bar{\Phi}})^\dagger \star \widehat{Q}_L^{(j)}) \Big)$$

$$- \sum_{i,j=1}^{3} G_d^{ij} \Big((\bar{\widehat{Q}}_L^{(i)} \star \rho_Q(\widehat{\Phi})) \star \widehat{d}_R^{(j)} + \bar{\widehat{d}}_R^{(i)} \star (\rho_Q(\widehat{\Phi})^\dagger \star \widehat{Q}_L^{(j)}) \Big) \Big),$$

where $\star$ is the star product, $\bar{\Phi} = i\tau_2 \Phi^*$ and $\rho(\hat{F})$ denotes the representation in the enveloping algebra of the field $\hat{F}$ (see [11] for details). The matrices W^{ij}, G_u^{ij} and G_d^{ij} are the Yukawa couplings. Note that in (2) we have not yet developed the fields in the enveloping algebra. The action (2) has the standard model as a limit for $\theta \to 0$:

$$\begin{aligned}
\hat{S}_{NCSM} \quad = \quad & \int d^4x \sum_{i=1}^{3} \overline{\Psi}_L^{(i)} i \not{D} \Psi_L^{(i)} + \int d^4x \sum_{i=1}^{3} \overline{\Psi}_R^{(i)} i \not{D} \Psi_R^{(i)} \\
& - \int d^4x \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \int d^4x \frac{1}{2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} - \int d^4x \frac{1}{2} \mathrm{Tr} G_{\mu\nu} G^{\mu\nu} \\
& + \int d^4x \Big((D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \Big) \\
& + \int d^4x \Big(- \sum_{i,j=1}^{3} W^{ij} \Big(\bar{L}_L^{(i)} \Phi e_R^{(j)} + \bar{e}_R^{(i)} \Phi^\dagger L_L^{(j)} \Big) \\
& - \sum_{i,j=1}^{3} G_u^{ij} \Big(\bar{Q}_L^{(i)} \bar{\Phi} u_R^{(j)} + \bar{u}_R^{(i)} \bar{\Phi}^\dagger Q_L^{(j)} \Big) \\
& - \sum_{i,j=1}^{3} G_d^{ij} \Big(\bar{Q}_L^{(i)} \Phi d_R^{(j)} + \bar{d}_R^{(i)} \Phi^\dagger Q_L^{(j)} \Big) \Big) + \mathcal{O}(\theta).
\end{aligned} \tag{3}$$

Using the quantization and renormalization methods presented in [12] we compute the β-functions of this model at the one loop approximation. We find:

$$\frac{\partial}{\partial \mu} \alpha_i(\mu) = \frac{1}{2\pi} b_i^{NC} \alpha_i^2(\mu), \;\; i \in \{1,2,3\} \tag{4}$$

with

$$b_i^{NC} = \begin{pmatrix} b_1^{NC} \\ b_2^{NC} \\ b_3^{NC} \end{pmatrix} = \begin{pmatrix} -22/3 \\ -22/3 \\ -11 \end{pmatrix} + N_f \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}, \tag{5}$$

where $\alpha_i =$ where g_1 is the U(1) gauge coupling, g_2 the SU(2) one , g_3 the SU(3) one, $N_f = 3$ is the number of family and $N_{Higgs} = 1$ is the number of Higgs bosons. If we compare the b_i of the standard model on a noncommutative spacetime to that of the standard model on a commutative spacetime

$$b_i^{SM} = \begin{pmatrix} b_1^{SM} \\ b_2^{SM} \\ b_3^{SM} \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_f \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}, \tag{6}$$

we see that the only difference is the factor $-22/3$ for the U(1) gauge coupling that comes from the nonabelian like term in the U(1) noncommutative gauge boson interaction.

We shall now study the grand unification of the gauge couplings. Clearly the running of the beta-function of the U(1) sector is in contradiction with experiment. The noncommutative parameter θ thus has to be space-time dependent in other words energy-momentum dependent. It has been shown how to formulate Yang-Mills theories on a space-time with an energy-momentum noncommutativity [13]. We shall not go into these details and treat the scale dependence of θ as a threshold effect. We shall assume that $\theta^{\mu\nu} = 0$ for $\mu < \Lambda_{NC}$ and $\theta^{\mu\nu} \neq 0$ for $\mu \geq \Lambda_{NC}$, i.e. the noncommutativity of spacetime can only be probed when one goes to short enough distances. We thus have one free parameter Λ_{NC}. We know from experimental bounds that $\Lambda_{NC} > \mathcal{O}(1 \text{ TeV})$ [14], however if spacetime noncommutativity is responsible for the unification of the gauge couplings of the standard model we will see that the typical scale for spacetime noncommutativity is much higher and out of reach of future colliders.

Taking the following input values [4] $\alpha_1(M_Z) = 0.0168$, $\alpha_2(M_Z) = 0.03322$ and $\alpha_3(M_Z) = 0.118$, we find that if we assume that the scale for spacetime noncommutativity is 8.2×10^{12} GeV, the three gauge couplings of the standard model unify at a scale of $\Lambda_u = 2.3 \times 10^{17}$ GeV and the unified gauge coupling α_u is equal to 0.0208 at the unification scale. Grand unification within the noncommutative setting avoids problems with the proton decay as the grand unification scale is much higher than in conventional unification models. Unfortunately, it seems hopeless to test such a long proton lifetime. As in any non-supersymmetric unified theory, e.g. SU(5), we expect the nucleon lifetime to be given by a dimension 6 operator which is suppressed by the unification scale squared i.e. the proton lifetime is given by:

$$\tau_p \propto \frac{\Lambda_u^4}{\alpha_u^2 m_p^5} = 1.8 \times 10^{41} \text{yr}, \tag{7}$$

where m_p is the proton mass. The present limit [15] on the proton lifetime is of the order of 10^{33} yr (this limit is obviously decay channel dependent). The result (7) is clearly out of reach for present or future experiments. However, noncommutative grand unification [16] is a viable alternative to supersymmetric unification and does not involve any new particle. It is also interesting to note that the scale 2.3×10^{17} GeV is not very far away from the Planck scale.

One of the main motivations to consider a noncommutative spacetime is that it introduces the notion of minimal length in quantum field theoretical models. A minimal length is a natural expectation of a unified theory of quantum mechanics and general relativity [17]. The effects of such a minimal length are expected to become relevant close to the Planck scale, which corresponds to an energy scale of the order of 10^{19} GeV, scale at which gravity would unify with gauge theories as in e.g. string theory. Is it very interesting to note that the scale for the gauge unification on a noncommutative spacetime is quite close to the scale where one expects gravity to unify with the other forces of nature

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